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The Degree of Mathematics Teachers' Application of Productive Thinking Skills in Jordan

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Abstract
The purpose of the study was to find out how often Jordanian math teachers used constructive thinking techniques. A questionnaire created by the researchers was one of the study's instruments. The study used a descriptive research methodology 2. There were 452 math instructors in the First Directorate of Education in the Zarqa Governorate; 257 of them were female and 195 were male. These teachers made up the study population. 1. Using a stratified random technique, 117 males and 132 females made up the study sample. The study's findings demonstrated a high degree of application and the greatest level of proficiency in the following areas: First: Urging students to self-regulate and monitor their level of knowledge in mathematics, Second: Asking students to interpret and express the mathematical problem in their own language. The study also showed that there were no statistical differences resulting from the variables of academic qualification and gender of mathematics teachers. The study recommended: Urging mathematics teachers to apply mathematical inference and mathematical justification skills, and activating postgraduate education programs.

Keywords
Creative thinking, Critical thinking, Mathematics teachers, Productive thinking

1. Introduction
Global developments and changes in the current era require developing teachers professionally, scientifically and educationally, especially mathematics teachers, and providing them with the necessary new skills that enable them to teach mathematics effectively and positively. The mathematics teacher plays an important role in achieving the objectives of the mathematics curriculum. In the classroom they engage with students through their daily teaching and answer their questions, develop their thinking skills, and improve their performance in mathematics.

In school, mathematics is a necessary subject. Educators have long viewed mathematics as a subject that requires memorization, and this mindset permeates their teaching. Thus, students may find it challenging to learn mathematics if they are not proficient in the subject (National Research Council, 2001). A math teacher's job is to guide pupils through the challenges. However, a teacher needs to have a firm understanding of teaching and learning in the classroom in order to assist pupils in becoming proficient mathematicians. A teacher must constantly adapt their approaches and tactics to fit changes in the curriculum (Mellegard & Pettersen, 2017).

Instructors who take into account their students' mathematical thinking are more successful in establishing student-centered learning environments, according to a number of projects implemented with the aim of assisting mathematics teachers' professional development. People with mathematical thinking are better able to learn both autonomously and independently and to see the value of applying knowledge and abilities (Isoda & Katagiri, 2012). The first step in mathematical thinking is for students to observe the things in their environment and try to understand how those objects relate to one another (Tall, 1995).

The development of thinking abilities has taken center stage in mathematics education (Hurst & Hurrell, 2016). The ability to think is essential for overcoming obstacles in life. This comprises the capacity for critical and creative thought as well as problem-solving abilities (Kaleiloglu & Gulbahar, 2014). Both analytical and creative thinking are

necessary for productive thinking (PT). Thinking is done in two stages: first, creatively to come up with the greatest options and solutions; second, critically to assess these options and solutions and select the best and most relevant one (Think, 2012: 2).

The teaching and learning processes aim at improving and developing all different thinking skills, including PT, so that learners can apply them in various daily lives in a way that differs from the situations in which they were acquired. PT plays a role in teaching all curricula, where there is a strong relationship between PT and teaching strategies (Al-Tamimi, Ghanim, & Farhan, 2023).

Frihtemer (1945) explained that PT begins with a deep understanding of the structure of the problem, which will help clarify basic characteristics to answer it in a true and correct way that is appropriate to the fundamental needs of the problem. PT is appropriate to the situation in which it is applied, as it involves moving from a state of confusion. In some strange issues from the basic characteristics to a new situation in which everything is around a clear, logical and appropriate issue, and the essence of the process is a kind of reorganization, that is, moving from meaningless concepts to clear concepts with clear meanings (Erika, Savardi ,& Ivana, 2015).

The careers of the Future, Five of the ten talents that have been identified for the youth of the current fourth industrial revolution—complex problem solving, critical thinking, creativity, emotional intelligence, and judgment and decision making—are closely tied to thinking. It demonstrates how critical thinking is to the educational process. It's time to modernize the way we educate and learn to think in the classroom, especially by utilizing PT based on reproductive thinking, which encompasses the majority of thinking styles (Ashutosh and Kamakshi, 2020).

According to Sternberg (2005: 189), PT is "a mental activity based on three interrelated mental skills: the analytical skills that serve as a means of diagnosing and criticizing useful ideas, the practical skills that provide the capacity to persuade others of the worth of ideas, and the creative skills that allow an individual to view problems in a novel light." These abilities interact to be highly-promoted concepts with cognitive, contextual, and psychological factors as well as thinking styles and motivation, depending on the choices made by the individual.And Hurson (2008:241) definition : “A type of thinking that generates new, difficult ideas and is important for innovation, growth, capacity building, and differentiation. It helps the student to think and work better. Thinking is organized, disciplined, and repeatable, which leads to improving the student’s thinking.” "Creative and innovative".

In other words, PT is the cognitive ability to refine one's creative work with critical thinking to give it power and value. It is also the ability to plan, reason logically, analyze, synthesize, evaluate, and make decisions to arrive at a solution to the problem. PT is a teaching method that encourages students to answer arithmetic problems using their own logic and reasoning abilities (Murdoch, English, Hintz, & Tyson, 2020). Students build confidence in their problem-solving skills and obtain a better understanding of conceptual math through this process. The objectives of productive struggle instruction include helping pupils acquire comprehension, sharpen their critical thinking abilities, and become self-sufficient problem solvers (Amidon, Monroe, Rock, & Cook, 2020; Granberg, 2016).

According to researchers, PT is an essential and organic component of the mathematical learning process (Russo, Bobis, Downton, Livy, & Sullivan, 2021). Math is meant to be applied in a range of contexts and to equip pupils for life beyond the classroom. Students gain self-learning experiences through challenging themselves intellectually. PT instruction enhances their capacity for self-direction and motivates children to apply this knowledge to other tasks (Lemley, Ivy, Franz, & Oppenheimer, 2019).

PT refers to a pattern of thinking that related to mathematics and allows the learner to unleash the ideas in their minds for the production and generation of novel ideas (Adshead, Thacker, Fuldauer, and Hall, 2019). PT also helps the learner to gain insight into the structural composition of the situation and access to understanding to ensure the significance (Jawad, Raheem, and Majeed, 2021).

The application of PT to the process of learning can help the learner to have a better understanding of knowledge content. For one to be able to think in this manner, he/she must be able to create a picture of the situation through imagination and plan. More so, this kind of thinking also needs reasoning, problem-solving, decision-making, and judgments. It also involves the generation of ideas, rather than the mere analysis of arguments (Majeed, Jawad, and ALRikabi, 2021). students should have experiments and they able to plan and run their own experiments to test their own ideas and learn more about science (Husnaini & Chen, 2019).

The highest point of problem-solving intelligence, PT is a sign of success in the workplace, in the classroom, and in interpersonal interactions. Students can investigate facts, spot patterns or relationships that are connected, and apply conceptual or inductive thinking as well as creative reconsideration while solving problems. Following that, students receive training on how to find original solutions and implement them to produce useful outcomes (Nizaruddin, Sutrisno, Yanuar, & Muhammad, 2019).

Jawad (2022) indicated according to psychologists, PT can only be achieved through two major requirements, which are creative thinking and critical thinking. Critical thinking: this type of thinking is a thinking pattern used by individuals when there is a need for them to create a judgment or express an opinion regarding a topic or an issue. Second: Creative thinking: this is the kind of thinking that produces solutions or new ideas that are completely different from the usual ones. Through this kind of thinking new ideas emerge. This study adopted that the definition of PT is: “the thinking resulting from the beginning of the ability to possess creative thinking skills and reach critical thinking skills”.

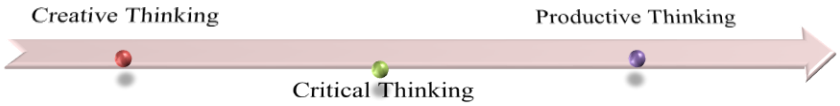

Fig. 1 PT, the beginning of possessing creative thinking and reach to critical thinking

The following is how Munandar (2002) characterizes the qualities of creative thinking abilities: 1. Fluency, or the capacity to create ideas, find solutions to issues, and offer numerous instances of a concept in specific contexts. 2. Flexibility, which includes coming up with ideas, offering responses in a variety of ways, utilizing a range of completion techniques, providing concept-related examples, and coming up with alternate solutions to several problems. 3. Originality is the capacity to create fresh ideas and unconventional, novel methods of expressing concepts that are novel, distinct, or distinctive. 4. Elaboration is the capacity to provide a thorough explanation, enhance, and develop a concept or product, add or itemize in detail of the situation so that it becomes more attractive, or answer specific mathematical situations. As shown in Figure 2.

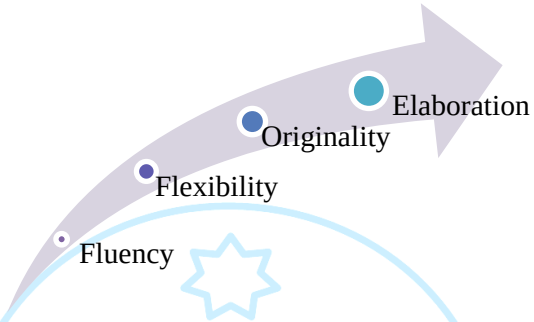

Fig. 2 Creative Thinking Skills.

According to Facione (2011), the following abilities are indicators of mathematical critical thinking ability:

1. Interpretation, or the capacity to comprehend and articulate the problem's meaning.
2. Analysis, which is the capacity to recognize and draw conclusions about connections between claims, queries, ideas, and explanations.
3. Inference, or the capacity to recognize and acquire the components required for a conclusion.
4. Evaluation is the capacity to ascertain the veracity of claims and representations and to discern the connections among questions, concepts, and descriptions.
5. Explanation, or the capacity to present and defend arguments supported by conceptual, methodological, logical, and contextual evidence.
6. Self-regulation, or the capacity to keep an eye on one's cognitive activity, is a key component of problem-solving techniques, particularly when using the analytical and evaluative skills. As shown in Figure 3.

Fig. 3 Critical Thinking Skills

Through the previous literature review, there are some studies that focused on The degree of mathematics teachers' applications of PT skills, including: The study Al-ajmi and al-ajmi (2023) aimed to identify to what extent social studies teachers in public schools in Kuwait employ PT skills. Similarly, The study used the descriptive approach by distributing a questionnaire consisting of 24 items, and answering it to a sample of 298 teachers who were selected by a stratified random sample from public schools in the Al-Ahmadi educational district in the State of Kuwait. the study revealed that social studies teachers employed PT skills to a high degree from their perspective. The findings also revealed that there were no statistically significant differences in the degree of practice in terms of gender and educational qualification.

Al-Zuhairy and Khaleel, 2022. The study seeks to find out (PT) among fourth-grade math students. To this end, the researcher created a PT in mathematics consisting of (34) paragraphs, which she verified. Of these, (9) essay paragraphs represent creative thinking skills (fluency, originality, flexibility) and (25) objective paragraphs represent critical thinking skills (interpretation, imposing assumptions, evaluation of arguments, deduction, and conclusion). The research sample consisted of 333 male and female fourth scientific students from the middle and secondary schools of the General Directorate of Anbar Education in the city of Ramadi. The exam was given to the sample by Psychometrics, and

the results showed that fourth scientific students' PT level is low and that there are statistically significant disparities in the mean scores of students in the mathematics PT test favoring male students.

Jawad (2022) aims to determine whether learners' capacity to make connections between mathematics and other sciences, as well as between their everyday lives and their capacity for constructive thought, is correlated with their ability to think mathematically. This is accomplished by means of two statistical tests, one of which was utilized to ascertain the learners' mathematics connection skills and the other of which was used to gauge the students' PT. For the study, a sample of 300 Baghdadi fourth-grade science students was used. The analysis' findings showed a favorable correlation between PT and mathematical connection ability.

The studies by Nizaruddin, Sutrisno, Yanuar, and Muhammad (2022) sought to characterize the PT of aspiring math teachers in answering limit function problems using the epsilon-delta concept. Descriptive qualitative research from PGRI Semarang's University of mathematics education research program was employed in this study. The two stages that were conducted were: providing self-regulated learning questionnaires and providing test questions requiring critical and creative thinking. The approach of triangulation, which compares written test results with interview data, yields credible research results. It shows that children who practice self-regulated learning are extremely creative and have critical thinking skills. They also exhibit the following mental habits: can write given facts—that is, facts expressed as symbols from the definition of the limit function—identify test problems thoroughly and write settlement strategies; find facts, data, and concepts and connect them to create problem-solving designs, particularly when looking for delta values; and correctly use algebra when looking for delta values. Despite the lack of information in the responses or delta values acquired, there is a lack of systematic algebraic computations and manipulations to demonstrate that the chosen delta produces a value for the limit function less my limit value range from Epsilon. While there is always room for improvement, kids that exhibit self-regulated learning generally possess PT abilities, particularly when it reaches to critical thinking.

Determining the degree of PT proficiency among middle school biology teachers was the aim of Abdul Amir's 2021 study. All biology teachers comprised the research population, and 200 teachers of all specializations were included in the sample, which was chosen at random. There were 100 male teachers and 100 female teachers. The PT Scale was created using a descriptive technique and comprises fifty-five items split into seven domains. The research objectives were met. Aesthetic validity and construction validity were then used to verify exam validity. The researcher used statistical analysis software (SPSS). The research sample of biology teachers had a modest degree of productive thinking, according to the findings. The differences in the study sample's average PT level forecasts, which benefited the teachers, were explained by the gender variable.

The goal of Naji, Alzobai, and A-Qaisi's (2021) research was to develop a training program based on self-organized learning methodologies for physics teachers and how it would affect their PT and technological enlightenment. There are 32 teachers and schools in the sample; 16 teachers are part of the experimental sample and 16 teachers are part of the control sample. The instruments, which included the 60-item Enlightenment Technology Scale, were ready. The PT Test, which has two domains—the Critical Thinking Field with fifteen items and the Creative Thinking Domain with nine items—was created by researchers. According to various self-organized learning methodologies, the results demonstrated that the proposed training program fits the students' training demands with exceptional effectiveness, enabling them to raise their level of teaching and educational technological performance and accomplish their targeted goals. The training plays a significant role in helping students transfer knowledge and cultivate a growth mindset around self-learning.

The following research question was used by Maurina, Richard, and Selcen Guzey's (2020) study to examine the design conversations of sixth grade students: How sixth-grade students use several ways of thinking to solve a science unit's design-based problem. Utilizing the analytical framework for PT developed by Gallagher and Aschner in a qualitative and descriptive case-study methodology. According to the findings, students use a range of thought processes when having design conversations. Students used Cognitive Memory, Divergent Thinking, and Evaluative Thinking when they planned their first design. This is to be expected as when students start to defend their design choices, they will need to recollect scientific facts and make conjectures. Since they assessed and provided justification for their design choices, they used more higher order modes of thinking. These results offer valuable information about productive thinking and conceptual performance teaching methods.

The research conducted by Bature and Atweh (2020) examined the ways in which math teachers reflected on the framework of Productive Pedagogies and its role in promoting effective classroom instruction. The four math teachers' reflections were discussed using two research objectives. The study's design was decided upon as a qualitative case study. Data collection methods for the study included classroom observations, research journals, and reflective interviews. The narratives approach to data analysis was employed to analyze the obtained data. With the help of the researcher and successful reflective meetings with their peers, the math teachers were able to progressively transition to the new framework as a result of the study. Teachers of mathematics reported that the framework had helped them become more self-assured, optimistic, and engaged in their profession. It promoted efficient teamwork and established a trusting environment between math teachers and their pupils. The study's conclusions indicated that, in order to enhance math teachers' instruction, Nigerian classrooms should implement the Productive Pedagogies framework.

The goal of Yanuar, et al. (2019) was to describe how PT students solved algebraic problems. This study's focus is on selecting four participants with various PT. A self-regulated learning questionnaire is used in the first instrument,

while tests and interviews are used in the second. The findings showed that students who have high levels of self-regulated learning are highly critical thinkers who are also creative thinkers. Students who possess the capacity for productive thought exhibit the following characteristics: They can accurately describe what is known and what has been questioned in the question, they can write facts clearly, they can accurately identify problems, they can write mathematical models, and they can perform calculations and verify that everything is done correctly. They provide multiple pertinent ideas and come up with different approaches to solving the problems. They also provide detailed, intricate responses, draw accurate conclusions, and perform precise calculations and double check their findings using the problem's existing information.

Regarding Abdullah's (2018) research, its goal was to determine the PT proficiency of Iraqi middle school instructors. The study used the descriptive survey approach, and the researchers created a 20-item questionnaire for PT. It was used on 179 instructors, both male and female. The findings indicated that middle school teachers in Iraq had a medium degree of physical therapy. The gender variable did not appear to have an impact on estimates of the amount of PT in the results.

The goal of the studies by Abdullah, Mokhtar, Halim, Farzeeha, Tahir, and Kohar (2017) was to determine the degree of expertise and application of higher-order thinking skills (HOTS) among math teachers at a secondary school in the Terengganu area. The study contrasted respondents' demographic characteristics with features of curriculum, pedagogy, and assessment. It employed a descriptive survey method with 196 respondents and a quantitative methodology. Additionally, the Multivariate Analysis of Variance Test (MANOVA) and Pearson correlation were used for inferential analysis. The results demonstrated that the assessment component's degree of practice and understanding was the lowest. Additionally, there was a correlation between each aspect's level of HOTS practice and knowledge. Based on demographic variables including gender, there were notable disparities in the degree of expertise and application of HOTS.

The Chang (2016) study aimed to ascertain the extent to which instructors in secondary schools employ PT knowledge. The descriptive technique was used in the study, which included 68 male and female teachers from the United States. The study's objectives were met by creating a questionnaire consisting of twenty-eight items, and the results showed that secondary school instructors made extensive use of PT talents across all subject areas. The findings showed that there was no statistically significant difference in the use of PT skills by secondary school instructors based on gender or academic background.

All previous studies that looked into how much math teachers in Jordan can use constructive thought processes did so using the descriptive technique. Previous studies have also looked at the effect of gender and the academic qualification variable, as shown by Chang (2016), Al-ajmi and al-ajmi 2023, and Al-Zuhairy & Khaleel, 2022. Therefore, the goal of this study was to determine the degree to which Jordanian math teachers are proficient in applying productive thinking strategies and to look into the implications of two important factors: gender (male or female) and academic standing (bachelor's degree or less, high diploma or above).

In Jordan, mathematics is considered one of the essential and important academic subjects and is taught at many educational levels. One of the main goals of mathematics education is to help students develop their critical thinking abilities, especially in the subject of mathematics. Only by providing math instructors with the necessary preparation and critical thinking skills to do their jobs effectively will this goal be achieved. Despite Jordan's efforts to develop math curricula since 2019, the need to comprehend the credentials and proficiencies of math teachers has increased. This is especially true in terms of the varieties and powers of mathematical reasoning. especially in the field of constructive mathematical reasoning, which combines the capacity for both critical and creative thought.

Research has shown that students have low levels of mathematical reasoning, as demonstrated by the following studies: Al-Harbi and Al-Mu'tham, 2014; Al-Qarni, 2019). Because mathematics courses are so important to Jordanian education, the study sought to ascertain the extent to which math teachers in that country are capable of productive thought. The function of math instructors in the learning process. The research specifically aimed for answers to the following queries:

1. What is the degree of mathematics teachers' application of productive thinking skills in Jordan?
2. Are there statistically significant differences (A = 0.05) in the degree of mathematics teachers' application of productive thinking skills in Jordan according to the variable of gender and academic qualification of mathematics teachers?

2. Significant Study

The importance of this study derives from the educational reforms that the educational system in Jordan is constantly working on, especially in the field of preparing and qualifying teachers, and determining the degree of knowledge and skills they possess. Accordingly, the study attempts to verify the degree to which mathematics teachers possess productive thinking skills in Jordan, which is reflected in their performance in teaching mathematics effectively. It is one of the few studies focused on this field. In addition to providing scientific research with a questionnaire prepared by researchers to determine productive thinking skills. As a result, the results of the study help mathematics teachers to qualify themselves or participate in qualification courses that increase their competence in productive thinking. It also helps curriculum planners and educational leaders to identify the skills that mathematics teachers possess, in order to benefit from them in providing all the means and capabilities that mathematics teachers need.

This study was limited to all mathematics teachers who teach mathematics for the basic stage in all government schools affiliated with the First Directorate of Education in Zarqa Governorate in Jordan, in the second academic year of 2023-2024 AD. The number of mathematics teachers in that directorate reached (452). The population was also limited to a questionnaire prepared by the researchers in light of the study's literature, and the study attempted to verify the degree to which mathematics teachers possess productive thinking skills in Jordan. The study was limited to answers given by mathematics teachers according to gender (male, female) and academic qualification (bachelor's degree or below, high diploma or above).

3. Study Methodology

In their study, the researchers followed the descriptive research methodology, in order to reach the results accurately and clearly, by distributing a questionnaire aimed at determining the degree to which mathematics teachers possess productive thinking skills, and calculating frequencies and averages of mathematics teachers' response values. Descriptive research is used in field research, in addition to the size of the population. The study included (452) teachers, and Abu Saleh & Awad (2012) indicated that quantitative data can be generalized to all members of society, and that the results of the study are very reliable.

2.1 Study Population and Sample

The study population consisted of all mathematics teachers who teach mathematics for the basic stage in all government schools affiliated with the First Directorate of Education in Zarqa Governorate in Jordan for the academic year 2023-2024 AD, where the number of mathematics teachers was (452) according to the directorate's data. Of them (195) males and (257) females, the study sample consisted of (249) teachers. Of them (117) are males, and (132) are females. They were selected according to the stratified random sampling method, and Abu Saleh and Awad (2012) indicated that the larger the sample percentage, the better its representation of the community.

2.2 Study Instruments

The researchers developed the study tool in light of the study literature, such as (Chang ,2016; Al-ajmi and al-ajmi 2023; Al-Zuhairy & khaleel, 2022), where the questionnaire consisted of (8) items, in order to determine the degree to which mathematics teachers possess For productive thinking skills in Jordan, the questionnaire was of the Likert Five Scale type, where the teachers' response to the questionnaire items was as follows: very high and was given (5) marks, high and (4) marks were given to it, moderate and (3) marks were given to it.) degrees, few and I gave her two marks, and very few and I gave her one mark. For the purpose of presenting the results, the level of thinking skills was graded according to the Likert Five Scale, and was defined as five levels: very high, high, moderate, little, and very little. Instructions for answering the questionnaire were written by the teachers, by placing a cross (X) in front of what they support for each paragraph of the questionnaire. The fifth level indicates that the degree of applications is very high, and the closer the teacher's answer is to the first level, the lower the degree of applications. The arithmetic averages for the degree of ownership were classified into five levels: very large, large, medium, little, and very little, according to the following equation:

(Highest value of answer alternatives - minimum value of answer alternatives) ÷ Number of levels.

(5-1)/5=4/5=0.8

This value is equal to the length of the category, and therefore:

From 1.00-1.8 is very low
From 1.8-2.6 a few
From 2.6-3.4 medium
From 3.4-4.2 high
From 4.2-5.00 is very high

2.3 Validity and Reliability of the Study Tools

To ensure the validity of the study tool, the researchers presented the initial image of the tool to a group of knowledgeable and experienced arbitrators with various academic qualifications who held master's and doctoral degrees. Eight arbitrators judged the study tool. Their comments included a set of suggestions, such as rephrasing some paragraphs, clarifying some paragraphs, and the paragraph should not be complex. A modified questionnaire was then prepared in accordance with the previous arbitrators' suggestions, and then it was presented again to five of the previous arbitrators. Their suggestions were limited and few. Then a questionnaire was prepared. The researcher completed the questionnaire in its final form.

To ensure the reliability of the study tool, it was verified using the test-retest method by applying the test, and re-applying it after two weeks to a group of (35) teachers outside the study, and then the Pearson correlation coefficient was calculated between their estimates the two times, as it was (0.81). The reliability coefficient was also calculated using the internal consistency method according to the Kuder Richardson-20 equation, reaching (0.78), and these values were considered appropriate for the purposes of this study.

2.4 Study Procedures

Evaluation of the research on the benefits of active thinking. The First Directorate of Education in the Zarqa Governorate of Jordan gave permission for the questionnaire to be sent to math teachers. There were 452 teachers in total according to

the study, and 249 teachers made up the study sample that was chosen using a stratified random approach. After confirming the questionnaire's psychometric properties (validity and reliability), it was constructed in its final form and adjusted using a five-point Likert scale before being sent electronically to the participants.

Following notification to participants that all information will be handled in strict confidence and used only for scientific research. Additionally, let the participants know that they are allowed to respond whatever they see fit to the studies prepared questionnaire. The study is carried out impartially and without bias, and any conclusions that come from it will not be influenced by the researchers. A little over 261 math teachers received the questionnaire; some were canceled, and just 249 were accepted for distribution. After being gathered, categorized, audited, and having its integrity guaranteed for statistical analysis, the data was input into the computer. The statistical analysis tool (SPSS) was utilized by the researchers to examine the data and produce the findings.

3. Findings

With two study questions that were addressed by the questionnaire created specifically for this investigation, the study's findings were provided.

The First Question

What is the degree of mathematics teachers' application of productive thinking skills in Jordan? To answer this question, the arithmetic means and standard deviations were calculated for each item on the application degrees of productive thinking skills for mathematics teachers. Table 1 shows the arithmetic means and standard deviations for the application degrees.

Table 1 Arithmetic means and standard deviations for scores on possessing productive thinking skills

Paragraph No.	Paragraph	Arithmetic Mean	Standard Deviation	Degree of application	Ranking
1	Encouraging students towards mathematical ideas and providing various answers to mathematical problems. And give many examples related to the mathematical concept in specific situations.	3.61	1.23	High	3
2	Encouraging students to generate mathematical ideas, use various strategies, and find alternative solutions.	3.53	1.28	High	4
3	Encouraging students to generate new and unique mathematical expressions, and to think unconventionally about ways of expressing new expressions.	3.42	1.36	High	7
4	Asking students to explain in detail, enrich and develop mathematical ideas, or add or elaborate on the situation so that it becomes more attractive.	3.23	1.37	Medium	10
5	Asking students to interpret, understand, and express the mathematical problem in their own language.	3.63	1.23	High	2
6	Urging students to analyze and extract mathematical relationships between expressions, questions, concepts, and procedures.	3.46	1.27	High	5
7	Encouraging students to Mathematical inference, identify and obtain the elements necessary for inference.	3.38	1.36	Medium	9
8	Urging students to evaluate their mathematical work, determine the reliability of the results, and logically reach the relationships between procedures and concepts.	3.45	1.26	High	6
9	Requiring students to provide mathematical justification and explain reasons in the form of conceptual, methodological, logical, and contextual evidence.	3.39	1.26	Medium	8
10	Urging students to self-regulate and monitor their level of knowledge in mathematics, especially in applying the ability to solve mathematical problems and evaluation.	3.65	1.23	High	1
	Overall	**3.47**	**1.29**	**High**	

It is clear from Table 1 that the overall degree of mathematics teachers' application of productive thinking skills in Jordan was high, and the Paragraph (10) came in first ranked, with a arithmetic mean (3.65) and a high degree. Followed it the Paragraph (5) with a arithmetic mean of (3.63) and a high degree. Paragraph (1), it came in third ranked with a arithmetic mean of (3.61) and a high degree.

The Second Question

Are there statistically significant variations (A = 0.05) between the degree to which math teachers in Jordan apply productive thinking skills based on their gender and academic background? The degree to which Jordanian mathematics teachers applied productive thinking techniques was measured using arithmetic means and standard deviations, taking into account the variables of gender and academic background. This is seen in Table 2.

Table 2 Arithmetic means and standard deviations for the degree of use of productive thinking skills by mathematics teachers in Jordan based on academic qualification and gender.

variables	Categories	Numbers	Arithmetic means	standard deviations
Gender	Male	117	3.47	.06
	Female	132	3.57	.05
Qualifications	Bachelor`s degree or less	204	3.44	.38
	Higher diploma or above	45	3.60	.08
Male	Bachelor`s degree or less	99	3.36	.51
	Higher diploma or above	18	3.59	.48
female	Bachelor`s degree or less	105	3.53	.53
	Higher diploma or above	27	3.62	.64

A two-way ANOVA test was conducted to show the effect of the variables of gender and academic qualification of mathematics teachers on the degree of applications, and Table 3 shows this.

Table 3 two-way ANOVA test showed the effect of the variables of gender and academic qualification Of mathematics teachers on degree of application.

Source of variance	Sum of squares	Degrees of freedom	Sum of means Value	F value	Significance
Gender	.33	1	.33	1.22	.26
Academic qualification	.92	1	.92	3.19	.07
Gender *academic qualification	.18	1	.18	.63	.42
Error	70.74	245	.28		
Total	3087.17	249			

Table 3 shows that there are no statistically significant differences in the degrees of mathematics teachers' application degrees of productive thinking skills in Jordan due to the variables of gender or academic qualification.

4. Discussing

4.1 Discussing the results of the first question

The results of the study clearly show how heavily math professors use critical thinking strategies. This outcome agreed with the research findings by Chang (2016) and Al-ajmi & Al-ajmi (2023). This is so because math teachers are aware of the importance of critical thinking in the study and instruction of mathematics. Mathematicians know that productive thinking helps students think creatively by using their mathematical knowledge, since creative thinking depends on the fluency, adaptability, and originality of expressing mathematical ideas. This is also a product of math teachers' dedication to imparting to their students critical thinking skills, such as interpretation, prediction, situational perception, and mathematical reasoning analysis. In contrast, the results of Abdullah's (2018) study indicated a moderate level of productive thinking.

And it is clear from the results of the study that Paragraph 10 came in first rank, which is "Urging students to self-regulate and monitor their level of knowledge in mathematics, especially in applying the ability to solve mathematical problems and evaluation.." This means that mathematics teachers realize the importance of urging students to self-regulate their abilities in learning mathematics. This makes students an active participant in the learning process by following a set of methods and procedures during different mathematics learning situations in order to achieve the desired learning outcomes and monitor their degree of development in learning mathematics. Self-regulation develops discussion skills, the ability to solve mathematical problems, and evaluates their performance in acquiring mathematics.

Paragraph 5 came in second rank, which is: "Asking students to interpret, understand, and express the mathematical problem in their own language.." This indicates the desire of mathematics teachers to ensure that students understand mathematical problems correctly. This enables mathematics teachers to identify misconceptions that students may make, and incorrect procedures that students may take while answering mathematical problems. In addition,

mathematics teachers enhance students' mathematical communication skills by giving students an opportunity to express mathematical problems in their own language by using their appropriate mathematical terminology.
Paragraph 1, "Encouraging students toward mathematical ideas and providing various answers to mathematical problems," finished in third place. and provide several instances of the mathematical idea in real-world contexts. This is because professors of mathematics are eager to have their pupils use mathematical concepts to provide multiple solutions to mathematical problems by using different approaches to solve the problem. Teachers of mathematics work to keep an eye on their pupils' grasp of mathematical ideas and how they apply them to different aspects of daily life. This serves to emphasize the value of mathematics in students' everyday lives.

4.2 Discussing the results of the second question

As the academic qualification represented the academic degree obtained by mathematics teachers (bachelor's degree or below, high diploma or above), the results demonstrated that there were no statistically significant differences attributable to the effect of gender and academic qualification on the degree to which mathematics teachers applied productive thinking skills. The teacher's gender (male or female) serves as a representation of gender.
The lack of an effect of academic qualification may be due to the fact that the academic qualification held by mathematics teachers did not play a role in causing differences in application degrees. This result agreed with Al-ajmi and al-ajmi (2023) and Chang (2016) studies. All mathematics teachers apply productive thinking skills in the same degree of learning and teaching mathematics. This indicates that there is no effect of graduate programs in increasing application degrees. This may be due to graduate programs and educational qualifications neglecting the applied aspect of productive thinking skills in mathematics education. Also, the nature of the content of university study programs and the number of university hours that teachers receive in their university studies do not focus on productive thinking skills and their impact on the education process.

The gender of mathematics teachers did not play a role in causing differences in application degrees. This result agreed with Al-ajmi and al-ajmi (2023) and Al-Zuhairy & khaleel (2022) and Chang (2016) studies, which indicated that the gender of the teacher did not play a role in determining the degree of productive thinking. This indicates that there is no effect of the teacher's gender, whether male or female, in increasing application degrees. This may be due to the fact that all mathematics teachers, regardless of their gender, apply productive thinking skills in teaching mathematics in the same degrees.

5. Conclusions

The research came to the following conclusions:

1. Teachers in Jordan are persuaded of the importance of using productive thinking techniques in mathematics instruction due to the high degree to which they utilize these skills.
2. The most fruitful ways for math teachers to think were:
   a. Encouraging students to monitor and self-regulate their mathematical knowledge, particularly when it comes to using their aptitude for problem-solving and assessment,
   b. Requesting that pupils analyze, comprehend, and articulate the mathematical issue in their own words.
3. Math teachers' least effective ways of encouraging pupils to think critically were:
   a. Asking them to elaborate on a situation to make it more appealing;
   b. Asking them to enrich and develop mathematical ideas; and c. Asking them to explain in detail.
4. Encouraging students to Mathematical inference, identify and obtain the elements necessary for inference.

6. Recommendations

Based on the study results, the study presents a set of recommendations:

1. Urging mathematics teachers to apply mathematical inference and mathematical justification skills, and encourage students to use them.
2. Activating postgraduate education programs and clarifying the role of productive thinking skills in mathematics education.

Acknowledging Biases and Pre-understandings

Asking mathematics teachers to answer the questionnaire prepared for the study objectively and clearly. Aside from any personal interests regarding the degree of their application of productive thinking skills in Jordan. Informing mathematics teachers that all evidence provided is for the purposes of scientific research. It will be treated with complete confidentiality. There are no conflicts of interest in this study.

Declarations

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